Efficient Tools for Managing Uncertainties in Design and Operation of Engineering Structures

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Abstract—Actual load, material characteristics and other quantities often differ from the design values. This can cause worse function, shorter life or failure of a civil engineering structure, a machine, vehicle or another appliance. The paper shows main causes of the uncertainties and deviations and presents a systematic approach and efficient tools for their elimination or mitigation of consequences. Emphasis is put on the design stage, which is most important for reliability ensuring. Principles of robust design and important tools are explained, including FMEA, sensitivity analysis and probabilistic simulation methods. The lifetime prediction of long-life objects can be improved by long-term monitoring of the load response and damage accumulation in operation. The condition evaluation of engineering structures, such as bridges, is often based on visual inspection and verbal description. Here, methods based on fuzzy logic can reduce the subjective influences.

Keywords—Design, fuzzy methods, Monte Carlo, reliability, robust design, sensitivity analysis, simulation, uncertainties.

I. INTRODUCTION

DURING design of engineering structures, machines and various appliances, the quantities important for their reliability and lifetime (load, material properties, geometry, action of environment etc.) are usually not known accurately. As a consequence, their actual values may differ from those used in design. In some cases this can result in worser function, shorter life, or even failure of the object. In the opposite case, the design can be uneconomical. Of course, many structures can be designed according to codes. However, the codes do not cover all situations. The use of advanced methods of analysis and design can lead to more economical and also safe constructions.

The right choice of appropriate methods depends on the knowledge of possible causes of uncertainties and of the consequences of deviations of input quantities from nominal values. The main causes are: 1) random character of input quantities, 2) insufficient knowledge of input values, e.g. due to limited amount of experimental data, 3) simplifications and shortcomings in the computer models, 4) changes of material and other properties or load level during long time, 5) human errors (negligence, intention), and 6) unpredictable events (collision with another object or natural catastrophes).

This paper shows eficient tools for the mitigation of some of these problems. The tools can be divided into those suitable for the design stage and those for operation.

II. RELIABILITY METHODS FOR DESIGN STAGE

Reliability and safety of an engineering object are mostly formed in the design stage. Every design occurs in three phases: 1) proposal of conception, 2) determination of parameters, and 3) prescription of tolerances. In the following section, reliability methods suitable for individual phases will be explained.

A. Proposal of Conception

When specifying the basic arrangement of a structure, one should be aware how it could fail, and take measures for improvement, as this is much more effective in the design stage than later in operation. Two procedures are very useful here: FTA and FMEA.

Fault Tree Analysis (FTA) looks for all possible ways the structure could fail. From each such „top-event" it goes „down" to simpler components, and tries to find all initial causes of failures. The relationships between individual structural members and their influence on the total reliability can be expressed by a reliability block diagram, which is later useful in the calculation of total probability of failure and in allocation of the allowable failure probabilities to individual parts. FTA has also been incorporated into international standards [1].

During the design stage, also a general philosophy for ensuring the reliability and safety must be chosen. For critical failures, with fatal consequences, the *fail-safe concept* is suitable, which tries to avoid them by doubling the critical members or using other kind of redundancy. This can influence the concept of the construction and the calculation of failure probability.

Failure Modes and Effects Analysis (FMEA) is a systematic procedure for revealing all possible causes of future problems and for avoiding or mitigating the most dangerous of them. FMEA is done as soon as the basic concept and parameters of the construction have been defined [2]. A team for the FMEA, established from the specialists for design, building and operation, strives to reveal all thinkable failure modes of individual components or processes, and to find their consequences for the object. Each failure mode is written down into a special form and assigned three numbers. The first number (S) characterizes the severity (consequences) of the failure, the second one (P) characterizes how often or how probably this failure can occur, and the third number (D) characterizes the probability of its early detection. Each

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number ranks between 1 and 10 (1 is the best and 10 the worst case), and its assigning to the particular case is a matter of team consensus. The product of all three numbers, called Risk Priority Number,

$$\text{RPN} = \text{S} \times \text{P} \times \text{D}, \tag{1}$$

characterizes the general siginificance of the pertinent failure mode. Then, the total RPN for the structure is calculated by summing up the risk priority numbers for all failure modes. In the next step, corrective actions are proposed. Usually, one only aims at failure modes with the highest RPNs or with the most dangerous consequences (loss of lives, high ecological damages, etc.). With the measures proposed, new RPNs are calculated. The effect of FMEA can be assessed by comparing the new RPN for the whole structure with the original one. RPN also enables ranking of the components according to reliability, and creation of so-called safety maps of the structure. Critical parts are then inspected more often and thoroughly („risk-based inspections").

The advantage of both methods (FTA and FMEA) is that they are simple, do not need special mathematical knowledge or tools, and are very effective.

B. Determination of Optimum Parameters

After the concept of the construction has been set down, it is necessary to determine all important parameters, dimensions, etc. Higher reliability can be achieved using *robust design*, i.e. design with low sensitivity of the output parameters to the deviations of input quantities from nominal values [3, 4]. This can be achieved by a suitable choice of nominal values of individual parameters (i.e. design point). Figure 1 illustrates this principle on an example with one input variable: point 1 is with high sensitivity, while point 2 is with low sensitivity, which is much better for reliability.
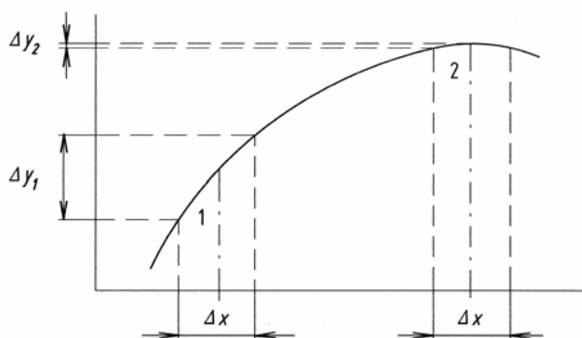

Fig. 1 Principle of robust design. 1 – point with high sensitivity, 2 – point with low sensitivity of y to variations of x

With several input quantities, *response surface* (Fig. 2) is used, which expresses the output variable y as a function of all input quantities, $y = f(x_1, x_2, \dots x_n)$. The analytical formula for y is known exactly only in simplest cases. Often, the response must be found by numerical solution (e.g. FEM). In such case, approximate expression for y is used, obtained by regression-fitting the response computed for several combinations of
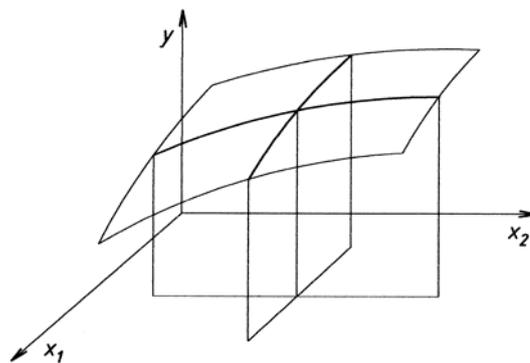

Fig. 2 Response surface for two independent variables, x_1, x_2

input parameters. The simplest form of a response function is a polynomial. In the vicinity of the design point, a polynomial of first or second order is usually suitable:

$$y = y_0 + \sum a_i(x_i - x_{i,0}) + \sum b_i(x_i - x_{i,0})^2 + \\ + \sum c_i(x_i - x_{i,0})(x_j - x_{j,0}) + \dots \tag{2}$$

The response surface can serve in searching for the optimum design point, in the sensitivity analysis, and in prescribing the tolerances to input quantities. Besides general methods for optimisation, also the procedures for design of experiments are suitable [3, 4].

C. Sensitivity Analysis

The aim of sensitivity analysis is to reveal the influence of individual variables and of simultaneous random variability of all input quantities on the variations in y, in order to find the design parameters with low sensitivity and to assign appropriate tolerances to input quantities [5, 6].

Direct influence of individual variables. The sensitivity of the response y to the variations of input quantity (e.g. x_i) is obtained from partial derivatives at the pertinent point,

$$c_i = \partial y / \partial x_i \approx \Delta y / \Delta x_i . \tag{3}$$

For linear approximation, the sensitivity coefficients c_i correspond to the constants a_i in (1). Further information is obtained from *relative sensitivities*,

$$c_{ri} = \frac{\partial y}{\partial x_i} \frac{x_{i,0}}{y_0} \approx \frac{\Delta y}{y_0} \bigg/ \frac{\Delta x_i}{x_{i,0}} , \tag{4}$$

where y_0 and $x_{i,0}$ are the values at the design point. Coeficient c_{ri} expresses the relative change of y, caused by 1% deviation of x_i from the nominal value $x_{i,0}$. For linear approximation, $c_{ri} = a_i(x_{i,0}/y_0)$. Note that the output deviation depends on the sensitivity c_i and the deviation of x_i.

Influence of random variations of input variables can be investigated using the expression for the scatter of a function of several random variables. For small scatter,

$$s_y^2 = \left(\frac{\partial y}{\partial x_1}\right)^2 s_{x1}^2 + \left(\frac{\partial y}{\partial x_2}\right)^2 s_{x2}^2 + \dots + 2\left(\frac{\partial y}{\partial x_1}\right)\left(\frac{\partial y}{\partial x_2}\right)\text{cov}(x_1, x_2) + \dots, \tag{5}$$

where s_{xi} is the standard deviation of x_i. For noncorrelated variables and linear approximation of y, the application of (5) on (2) gives

$$s_y^2 = a_1^2 s_{x1}^2 + a_2^2 s_{x2}^2 + \ldots + a_n^2 s_{xn}^2 + \ldots \; . \qquad (6)$$

The individual components, $s_{yi}^2 = a_i^2 s_{xi}^2$, give the scatter of y caused by random variations of i-th variable. The contribution of variable x_i to the total scatter s_y^2 is bigger for larger scatter of this variable (s_{xi}^2) and for larger sensitivity (a_i) of the output y to the changes of x_i. Division of (6) by s_y^2 gives the relative proportions of individual factors in the total scatter

$$1 = a_1^2 \frac{s_{x1}^2}{s_y^2} + a_2^2 \frac{s_{x2}^2}{s_y^2} + \ldots + a_n^2 \frac{s_{xn}^2}{s_y^2} + \ldots \quad . \qquad (7)$$

The influence of scatter of the individual input quantities can also be assessed by means of the ratio of the variation coefficient of the i-th variable and the variation coefficient of the output, corresponding to the scatter of this variable only,

$$\omega_i = \frac{v_y}{v_{xi}} = \frac{s_y}{y_0} \bigg/ \frac{s_{xi}}{x_{i,0}} \quad . \qquad (8)$$

Sensitivity analysis using simulation methods. The influence of random variability of input quantities can be assessed even without analytical expression for the response function – by means of probabilistic simulation techniques such as the *Monte Carlo*. This method is based on numerous repeating fictitious trials on a computer. In each trial, random value is assigned to each input variable, and the output quantity y is computed. Large number of trials gives the histogram of y (Fig. 3). This gives a general idea and can be used for the determination of average value of y or of extreme values that will be exceeded only with very small probability. The use of the Monte Carlo technique for reliability assessment of engineering components and structures is shown on many examples in [7]. Commercial software exists for these purposes, e.g. [8 – 10].
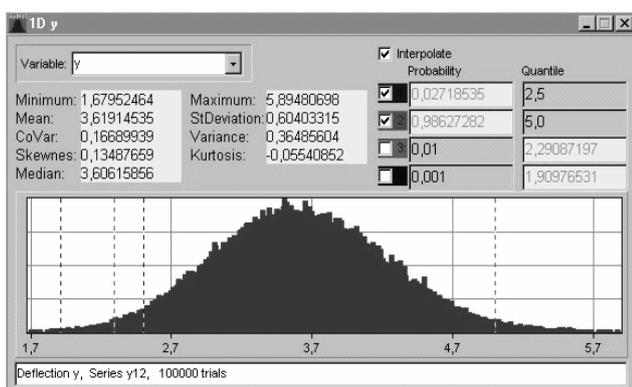

Fig. 3 Screen of the Monte Carlo simulation program Ant-Hill [8]

A simple sensitivity analysis by the Monte Carlo method consists of making m trials, with only one random variable (x_i), and then calculating the partial scatter s_{yi}^2 of y. Then, one can determine the ratios of variation coefficients or the sensitivity coefficients a_i ($= s_y/s_{xi}$) and the coefficients of relative sensitivity. A more detailed information is obtained if all input variables, $x_1, x_2, \ldots x_n$, are considered simultaneously as random quantities in the Monte Carlo simulations. The relative influence of individual factors can be obtained using (8).

The direct use of the Monte Carlo method is suitable for simple cases only. If the calculation of the response in one trial lasts seconds or more, the thousands of simulations would consume too much time. In these cases, the Monte Carlo analysis is faster if it is performed with a simple *response surface function* (2). Another method suitable for the analysis of random variability is *Latin Hypercube Sampling*. The definition range $\langle 0; 1 \rangle$ of the distribution function F is divided into m layers, and the response y is determined for m combinations of input variables. The $x_{i,j}$ values (i-th input variable, j-th layer), calculated from $F_j(x_i)$ values using the inverse probabilistic transformation F^{-1}, are chosen and combined randomly so that each value $x_{i,j}$ is used only once. The number of layers (and thus the number of simulations) is usually only several tens. The obtained y–values are used for the determination of statistical characteristics and for sensitivity analysis [11, 12].

D. Determination of Tolerances of Input Variables

If the variability of the output y is larger than allowed, it must be reduced. This can be accomplished by reducing the variance of input factors or their influence. Equation (7) shows which factors have the strongest influence. Very often, one factor prevails (for example x_k). As it follows from (6), the scatter of y can be reduced by reducing the standard deviation $s_{x,k}$ or by reducing the sensitivity of y to changes of x_k (coefficient a_k). Variance can be reduced by more accurate manufacturing or sorting out all parts, which are out of the tolerance limits. Sensitivity of y to the changes of x_k can be reduced by suitable choice of the design point (Fig. 1). If more input variables are involved, one must consider what changes will be most effective. With respect to various constraints and impossibility of changing some input quantities continuously, the optimal solution is usually found by comparing several variants. The method is described in detail with an application example in [6].

E. Reduction of Uncertainties using Bayesian Methods

The conclusions about some quantity can be more reliable by combining information from various sources (e.g. from similar components or structures). A classic problem in the probability theory is the determination of probability that an event „B" occurs after another event „A", which, however, can occur in several mutually excluding ways ($A_1, A_2, \ldots A_n$). Bayes theorem looks at the issue in the opposite way: „If the event B has occured, what is the probability that it was after (or due to) the event A_i?" This is the base for methods, denoted as Bayesian. An example of their application is nondestructive inspection: components are checked for cracks,

but the used device is not perfect. It classifies a defect correctly (as defect) only with probability 98%, while in 2% it (incorrectly) denotes the wrong part as good. On the other hand, the device marks 96% of good parts as good, but 4% classifies as with a crack. According to long term inspection records, 3% of all tested components contain cracks. The questions are: If the tested part was classified as „wrong" (i.e. with a defect), what is the probability that it is acutally: a) wrong, b) good? And what if the component was classified as „good"? The application of Bayes theorem (with the additional information of 3% defects) shows that the probability that a component, denoted as good, contains a crack, is 0.06%, and 99.94 that it is good. The probability that a part denoted as bad, is actually bad, is 43.1%, and 56.9% that it is good. [For example, the solution for the 0.06% case is 0.03×0.02/(0.03×0.02+0.97×0.96). For 43.1% it is 0.03×0.98/(0.03×0.982+0.97×0.04) = 0.4311.]

There are various Bayesian methods. Some of them are also used for continuous random variables, such as crack lengths or time to failure [13]. They are often approximated by Weibull distribution

$$F(t) = 1 - \exp\{-[(t - t_0)/\alpha]^\beta\} \quad ; \quad (9)$$

α, β and t_0 are parameters, which must be determined from tests or observations. Sometimes the amount of data is too low for obtaining reliable values of all parameters. However, one can use the fact that failures with similar mechanism have similar value of the distribution shape parameter β. If we have failure data from many similar objects with the same failure mechanism (e.g. fatigue of some steel brand), we may use their constant β also for the new case. The determination of the two remaining parameters α and t_0 is then more reliable.

Bayesian approach can also be used for updating parameters or quantiles of normal distribution using additional data; the procedure has been included into standards [14]. Further information about Bayesian methods for civil engineering structures can be found in [13,15].

More about any of the mentioned method (including information on software) can be found via Internet.

III. METHODS FOR MITIGATING UNCERTAINTIES IN OPERATION STAGE

Engineering structures in operation deteriorate gradually due to fatigue, corrosion and other effects of the load or environment. An accurate prediction of these processes and of the life-time is impossible especially for long-life structures, such as bridges. In this section, two methods for improvement will be described: computer-supported monitoring of load effects, and fuzzy methods for the evaluation of technical condition.

Computer-supported monitoring of load effects. Metal structures exposed to periodic load, such as bridges, suffer by fatigue. There are proven methods for fatigue assessment and for prediction of remaining time to failure, provided the load spectrum and history are known. Unfortunately, only in some cases the loads can be predicted accurately for a long period.

More accurate information is obtained by monitoring the loads and stresses in important parts of the structure. This can be done, e.g., by direct measurement via strain gauges fixed to the structure. However, a long term monitoring (years) needs that the strain gauges and all components in the measuring chain have very high reliability and long life and must be protected sufficiently against weather and mechanical damage.

An alternative approach, suitable, for example, for railway bridges, is based on computer simulation. The stresses in a structure can be calculated using the finite element method, provided the loads are well known. Today, basic information about loads can be obtained from the rail information systems. Railway companies store the data about the movement of all trains in the railway network. These data can yield the necessary information about the individual trains passing over a particular bridge: the types of locomotives and cars and the weights of transported goods. Together with the data about the weights of vehicles and their dimensions (axle distances), one can create the virtual load schemes for individual trains. At the University of Pardubice [16, 17], the pertinent method was developed, which consists of the following steps. First, finite element model of the bridge is created. Then, the influence line for internal forces and stresses at the investigated point is created by static analysis. Finally, train passage is simulated by moving the virtual load along this influence line. For this purpose, a computer program has been developed, able to calculate the time course of stresses, as well as to find the characteristic values with respect to the purpose of the analysis, e.g. the rain-flow sorting for fatigue assessment.

The proposed method has been verified by comparing the calculated stresses with those measured by strain gauges. The measurements and calculations were done for two steel railway bridges: a truss bridge and a plate-girder bridge, both over the Labe (Elbe) river. The stresses were measured using strain gages glued at various points of each structure (main girders, cross beams and stingers). The traffic was monitored 24 hours, with about 150 train passages over each bridge. The strain analysis was performed by a finite element code IDA NEXIS. The load models were created using the train data from the information system of Czech Railways, and processing them by a special computer program. Figure 4 shows very good agreement between the measured and calculated time course of stresses in one bridge. The agreement was good also in other tested cases. For the common train velocities, the quasistatic model was sufficient. The results are promising and indicate that this method could be used for the evaluation of bridge safety as well as for the assessment of accumulation of fatigue damage and of the remaining lifetime, the more so that it enables consideration of influence of the dead weight and thermal stresses from varying temperatures. More details about the method and computer models can be found in [16 – 18].
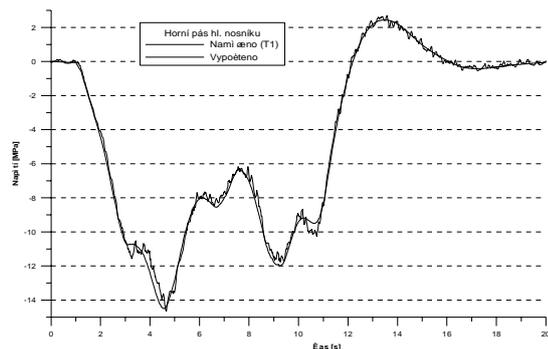
Fig. 4 Stresses caused by a passenger train in the main girder of a bridge [16]. Horizontal axis: time [s], vertical axis: stress [MPa]. Curve with undulations – measured, smooth curve – calculated

Fig. 5 Editor of a fuzzy inference system in Matlab [19]

Fuzzy methods. In inspections of civil engineering structures (e.g. bridges), only part of information has quantitative character, while some information is vague or „fuzzy" („the girders are very rusty", "there are many little cracks in the concrete wall", „the condition of central bridge span is relatively good", etc.). Information of fuzzy character is used if exact measurement is impossible or would be too expensive, or if it is common and sufficient. For example, when driving a car, one also does not work with accurate values, but with vague notions such as „far – near", or „fast – slow".

The need of working with vague quantities has led to the development of methods based on fuzzy-logic. They enable work with linguistic as well as numerical quantities, allow their combination and also the use of mathematical and logic operators (IF, AND, OR, THEN…). The application of fuzzy logic for evaluation of technical condition consists of three steps. In the first step („fuzzification"), so-called membership functions for individual input quantities are defined, which express in analytical form the interval and relevance of the term used (e.g. „small cracks"). In the second step, logic and mathematical operations are performed with the fuzzified input variables. In the third step (defuzzification), the resultant quantity is transformed to a sharp value, characterizing the overall condition („the damage degree is 4.3"), which can then be used for the decision about further operation or repair.

Today, commercial software exists for these methods (e.g. Fuzzy Logic Toolbox in Matlab [19], Fig. 5, or special SW).
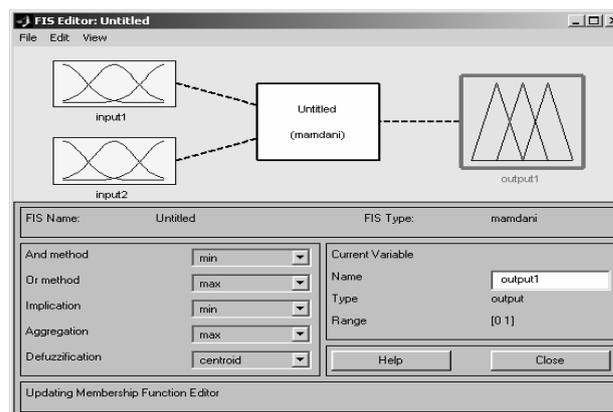
Thus, the main problem in practical applications is the preparation of input data and rules for the evaluation. For example, for bridges it means:

1) definition of parameters and criteria for the assessment (e.g. condition of the concrete plate, steel reinforcement, moulding, behavior during train passage, etc.),

2) definition of various degrees of deterioration or of characteristic response for the individual criteria (i.e. definition of membership functions),

3) assignment of the attributes to the individual criteria according to the actual state,

4) definition of rules for processing the input variables and for defuzzification of the result.

All this must be done in cooperation with experts. The advantage of the use of computer-supported fuzzy-logic methods for condition evaluation is the possibility of simultaneous considering a high number of criteria plus reduction of subjective influences in the judgement. More about these methods can be found in [20, 21], or in the thesis [22], devoted to bridges.

IV. CONCLUSION

Material properties, load and other quantities can differ from their nominal values. This influences the reliability, safety and performance of various engineering structures and appliances. The paper gave a brief overview of efficient methods for mitigating the unfavorable consequences of these deviations. Among the nonprobabilistic methods, the simple Failure Mode and Effect Analysis is very useful. If sufficient amount of data on randomly varying loads and properties is at disposal, probabilistic methods are suitable, including the numerical simulation techniques Monte Carlo or Latin Hypercube Sampling for complex cases. The effectivness of statistical inference can be increased by Bayesian approach, which combines information from various sources. Verbal (rather vague) characterization of technical condition can be processed using fuzzy methods. The information about load effects on a particular construction can sometimes be gained from information systems monitoring the traffic or operation.

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