Application of GA Optimization in Analysis of Variable Stiffness Composites

Nasim Fallahi, Erasmo Carrera, Alfonso Pagani

Abstract—Variable angle tow describes the fibres which are curvilinearly steered in a composite lamina. Significantly, stiffness tailoring freedom of VAT composite laminate can be enlarged and enabled. Composite structures with curvilinear fibres have been shown to improve the buckling load carrying capability in contrast with the straight laminate composites. However, the optimal design and analysis of VAT are faced with high computational efforts due to the increasing number of variables. In this article, an efficient optimum solution has been used in combination with 1D Carrera's Unified Formulation (CUF) to investigate the optimum fibre orientation angles for buckling analysis. The particular emphasis is on the LE-based CUF models, which provide a Lagrange Expansions to address a layerwise description of the problem unknowns. The first critical buckling load has been considered under simply supported boundary conditions. Special attention is lead to the sensitivity of buckling load corresponding to the fibre orientation angle in comparison with the results which obtain through the Genetic Algorithm (GA) optimization frame and then Artificial Neural Network (ANN) is applied to investigate the accuracy of the optimized model. As a result, numerical CUF approach with an optimal solution demonstrates the robustness and computational efficiency of proposed optimum methodology.

Keywords—Beam structures, layerwise, optimization, variable angle tow, neural network.

I. Introduction

IN recent years, Variable Angle Tow (VAT) composites can be used to design the lightweight structures with increased performance to use in aerospace applications [1]–[3]. The continuous variation of the stiffness properties obtained by curvilinear fibre path can provide considerable advantages in comparison with the straight composite laminates. In the preliminary design, buckling analysis is often a primary design criterion. It has been reported by a vast number of researchers that the capability of VAT plates under the buckling load [1], [4], [5]. Compared with the benefits provided by VAT, optimal design of VAT laminates requires to overcome the difficulties due to the higher number of design variables. The design of VAT laminates faces with a high number of variables in the layup sequence at each point of the structure. The optimization procedure in composite structures can be provided by the classical theories and definition of lamination parameters [6]–[8]. Classical approach benefit is that since the given lamination parameters are usually achieved by a set of fibre angles, they are capable to provide optimum fibre angles for a multiple-ply laminate. The problem of optimizing

N. Fallahi, E. Carrera and A. Pagani are with the Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Corso Duca degli Abruzzi, 10129 Torino, Italy (e-mail: nasim.fallahi@polito.it, erasmo.carrera@polito.it, alfonso.pagani@polito.it).

lamination parameters is limited depending on the initial solution than direct optimization of fibre orientation [9].

There is a vast literature on the subject of layerwise (LW) theory which is implemented in Carrera Unified Formulation (CUF) [10], [11], and in the form of 1D beam [12], [13], and also applied to VATs [14], [15]. Through different design optimization algorithms, evolutionary strategies like Genetic Algorithm (GA) have been used widely and suggested for optimizing composite structures [16]. GA is the most well-known and applicable meta-heuristic algorithm which for the first time was introduced by Holland [17] in 1975. Different types of problems can be solved by GA optimization methods [18], [19]. The purpose of this work is to fill the lack of research on LW theory through the optimization procedure which can present a robust, efficient and applicable design of VAT through CUF framework to find the maximum critical buckling load. Current research is capable to evaluate the optimization of fiber orientation angles in a VAT plate, layer by layer, accurately.

II. Linear Equations for VAT Laminates

A square laminate with $254 \times 254 \ mm$ is designed with the thickness of $0.15 \ mm$ for each ply of a 16-ply balanced symmetric laminate $[< T_0|T_1 >< -T_0| - T_1 > / < -T_0| - T_1 >< T_0|T_1 >]_s$ based on [20]. Mechanical properties of material are given by: E_1= 181 GPa, $E_2 = E_3$= 10.270, $G_{12} = G_{13}$= 7.170, G_{23}= 3.780 and ν_{12}= 0.28. The linear variation of fibre orientation angle is used [1], [2]. Fibre path can be designed for the curvilinear fibre path [21] which varies linearly along one axis, that in the current study is along the y-direction and can be written as:

$$\theta(y) = 2(T_1 - T_0)\frac{|y|}{a} + T_0 \qquad (1)$$

where T_0 is the fibre orientation angle at the centre of the plate, $x = 0$, and T_1 is the fibre orientation angle at the edges. a refers to the width of the VAT panel. Fibre orientation angle varied through the $y - axis$ and it showed by $\theta(y)$ to manufacture the entire ply. Note that in current study equal number of functions are used through both orthogonal directions x and y for all cases.

III. Numerical Approach

A. Carrera Unified Formulation

In the CUF framework, a beam is investigated by its cross-section which lies on the $xz-axis$ of a generic Cartesian reference system. The boundaries of the beam along the

$y - axis$ are limited to $0 \leq y \leq L$, where the L is the length of the beam. In CUF the displacement field for the beam structure can be expressed as a generic expansion of primary unknowns:

$$\mathbf{u}(x,y,z) = F_\tau(x,z)\mathbf{u}_\tau(y), \qquad \tau = 1,2,\ldots,M \quad (2)$$

where F_τ is an arbitrary cross-section expansion function over the $x,z - plane$, u_τ is the generalized displacements vector, and M refers to the term number of expansions. The kinematics of model can be modified according to the function F_τ in 1D CUF beam model which used Lagrange element as the expansion polynomials and denoted by $L9$. Lagrange polynomial expansions can formulate the quadratic higher-order kinematics. The L9 polynomial expansion is defined by the following kinematics [22]–[24]:

$$\mathbf{u}_x = F_1 u_{x1} + F_2 u_{x2} + \ldots + F_9 u_{x9}$$
$$\mathbf{u}_y = F_1 u_{y1} + F_2 u_{y2} + \ldots + F_9 u_{y9}$$
$$\mathbf{u}_z = F_1 u_{z1} + F_2 u_{z2} + \ldots + F_9 u_{z9} \quad (3)$$

where $F_1, F_2, \ldots, F_9$ are the nine Lagrange polynomials as the function on cross-section coordinates, and $u_{x1}, u_{y1}, u_{z1}, \ldots, u_{z9}$ are the displacement unknown variables through the $y - axis$ which represent pure displacement components at each root of the $L9$ polynomial set. The Lagrange expansions enable the laminate to be evaluated in the scheme of the LW approach, which is employed by defining a specific model for each layer. Consequently, the cross-section description describes separately in the laminate sheet and every single layer. Besides, LW enabled to increase the accuracy of distinguishing the mechanical behaviour in compared to the classical model based on Equivalent Single Layer (ESL) theory [25].

B. Finite Element Approximation

The finite element model (FEM) can be assumed along the $y - axis$ for discretization of structure which is approximated as:

$$\mathbf{u}(x,y,z) = F_\tau(x,z)N_i(y)\mathbf{q}_{\tau i} \qquad i = 1,2\ldots,K \quad (4)$$

where i is the index for number of nodes in the beam element, $q_{\tau i}$ is a vector of the FE nodal parameters and K is the number of nodes on the element.

C. Fundamental Nucleus

Based on Principal Virtual Displacement (PVD), the virtual internal work can be express as:

$$\delta L_{int} = \int_V \delta \epsilon^T \sigma dV \quad (5)$$

where V is the element volume, σ stands as the stress vectors and $\delta \epsilon$ refers to the virtual variation of strain which is presented as:

$$\delta \epsilon = \mathbf{b} \delta \mathbf{u} = \mathbf{b}(F_s(x,z)N_j(y))\delta \mathbf{q}_{sj} \quad (6)$$

where $\delta \mathbf{q}_{sj}$ is the virtual variation nodal unknown. By (2), (4) and (6) geometrical relations can be express as a linear form. Therefore, the virtual variation of internal work will be evident as:

$$\delta L_{int} = \delta \mathbf{q_{sj}^T} \underbrace{\int_V \mathbf{b^T}N_j(y)F_s(x,z)C\mathbf{b}F_\tau(x,z)N_i(y)dV}_{\text{Fundamental Nucleus}} \mathbf{q_{\tau i}}$$

$$= \delta \mathbf{q}_{sj}^T \mathbf{k}^{\tau sij} \mathbf{q}_{\tau i} \quad (7)$$

where $\mathbf{k}^{\tau sij}$ is CUF *Fundamental Nucleus* (FN) of the element stiffness of matrix k. The FN is a 3×3 matrix that present the cross-sectional function where $F_\tau = F_s$ for $\tau = s$ and shape functions is $N_i = N_j$, for $i = j$, which can be expand by using the indexes to be obtained the element stiffness matrix of any arbitrary refined beam model [23]. Based on the path function in VAT composites, each layer provides point-by-point continuous angle variations with different values. For VAT, the components of the FN make use of volume integrals. For the sake of brevity, only two terms of the FN are given in the following and the others can be given by permutations [22]:

$$k_{xx}^{\tau sij} = \int_V C_{22}F_{\tau,x}F_{s,x}N_iN_jdV + \int_V C_{66}F_{\tau,z}F_{s,z}N_iN_jdV$$
$$+ \int_V C_{44}F_\tau F_s N_{i,y}N_{j,y}dV;$$
$$k_{xy}^{\tau sij} = \int_V C_{23}F_\tau F_{s,x}N_{i,y}N_jdV + \int_V C_{44}F_{\tau,x}F_s N_iN_{j,y}dV; \quad (8)$$

where $\mathbf{C}$ is the stiffness coefficients of the elastic stiffness tensor and can vary within the computational domain; therefore, they must remain inside the integral of the FN. In the VAT structure, each fibre path can be defined as an arbitrary function, and the fibres follow the curvilinear pattern. Hence, in the domain of plate, C is no longer constant. Therefore, the integrals can be expressed in a unique form of the volume based on (8). In addition, the Gauss integration technique is applied in the framework of CUF. As a result, the material coefficients in VAT laminates can be considered in any specific Gauss point. Moreover, 1D CUF beam model is guaranteed a smooth approximation of the stiffness in the component in contrast with the finite element method [15], [22].

D. Buckling Formulation

For VAT composites, the *Tangent stiffness matrix* is given by the linearization of the virtual variation of the nonlinear internal strain energy $\delta(\delta L_{int})$:

$$\delta(\delta L_{int}) \approx \delta \mathbf{q}_{\tau i}^T \mathbf{k}^{\tau sij} \delta \mathbf{q}_{sj} + \int_V \delta(\delta \epsilon)^T \sigma^0 dV \quad (9)$$

where $\delta(\delta L_{int})$ is the sum of linear stiffness and virtual variation of work which is incorporated with initial stresses σ^0. Then, by utilizing (2), (4) and (10) and the Green-Lagrange nonlinear strain-displacement relations [26], the following formulations can be provided (see [23]):

$$\delta(\delta L_{int}) \approx \delta \mathbf{q}_{\tau i}^T \mathbf{k}^{\tau sij} \delta \mathbf{q}_{sj} + \delta \mathbf{q}_{\tau i}^T \mathbf{k}_{\sigma^0}^{\tau sij} \delta \mathbf{q}_{sj}$$
$$= \delta \mathbf{q}_{\tau i}^T (\mathbf{k}^{\tau sij} + \mathbf{k}_{\sigma^0}^{\tau sij}) \delta \mathbf{q}_{sj} \quad (10)$$

TABLE I
BEAM REFINEMENT IN CUF FRAMEWORK IN CONTRAST WITH
REFERENCE

Model	DOF	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
ABAQUS [20]	387205	13.62	21.62	35.40	54.46	56.01
CUF 10B3	43659	13.78	22.03	37.67	55.24	60.57
CUF 15B3	95139	13.61	21.69	35.94	54.51	57.65
CUF 20B3	166419	13.67	21.68	35.69	54.60	56.69

where $\mathbf{k}_{\sigma^0}^{\tau s i j}$ stands as the FN of the geometrical stiffness matrix:

$$\mathbf{k}_{\sigma^0}^{\tau s i j} = (\int_V \sigma_{xx}^0 F_{\tau,x} F_{s,x} N_i N_j dV + \int_V \sigma_{yy}^0 F_\tau F_s N_{i,y} N_{j,x} dV$$
$$+ \int_V \sigma_{zz}^0 F_{\tau,z} F_{s,z} N_i N_j dV + \int_V \sigma_{xy}^0 F_{\tau,x} F_s N_i N_{j,y} dV$$
$$+ \int_V \sigma_{xy}^0 F_\tau F_{s,x} N_{i,y} N_j dV + \int_V \sigma_{xz}^0 F_{\tau,x} F_{s,z} N_i N_j dV$$
$$+ \int_V \sigma_{xz}^0 F_{\tau,z} F_{s,x} N_i N_j dV + \int_V \sigma_{yz}^0 F_{\tau,z} F_s N_i N_{j,y} dV$$
$$+ \int_V \sigma_{yz}^0 F_\tau F_{s,z} N_{i,y} N_j dV)\mathbf{I} \qquad (11)$$

where the stress tensor is obtained by 9 components corresponding to 3×3 as an identity matrix $\mathbf{I}$. At the end, global matrices can be assembled in the classical FEM. The critical buckling loads are determined as those initial stress states σ^0, which render the tangent stiffness matrix singular; i.e., $| \mathbf{K} + \mathbf{K}_\sigma^0 | = 0$, [23].

IV. RESULTS

A simply supported (SSSS) boundary condition (BC) is applied on a symmetric square laminated plate. The two sides of the plate are under a compression load with $F = 1kN$ along the beam axis of y, for more details different analysis of VAT, see [27].

A. Direct Optimization GA

The aim of current research is to increase the first critical buckling load (F_{cr}) to improve the performance of the square laminate under the SSSS boundary condition under the buckling load. The buckling analysis has been done in CUF framework through the optimization procedure. For optimization, Genetic Algorithm (GA) is applied as a direct optimization in the MATLAB R2017b environment. In current problem, design variables for optimization procedure set on the T_0 and T_1 and are subjected to $0° \leq T_0, T_1 \leq 90°$ [28]. The objective function is set on to minimize the $1/F_{cr}$ (maximize the F_{cr}). Size of population and cross-over fraction is set on 50 and 0.8, respectively. The preliminary analysis is done through CUF model with 10, 15 and 20 beam element align to the $y - axis$ and compared with the reference in ABAQUS [20] for the VAT laminates with $[< 60°|15° >< -60°| - 15° > / < -60°| - 15° >< 60°|15° >]_s$. By a refinement of the beam elements in Table **??**, CUF showed a good convergence properties respect to the reference FEM solution [20]. The results obtained by CUF illustrated 8.86%,
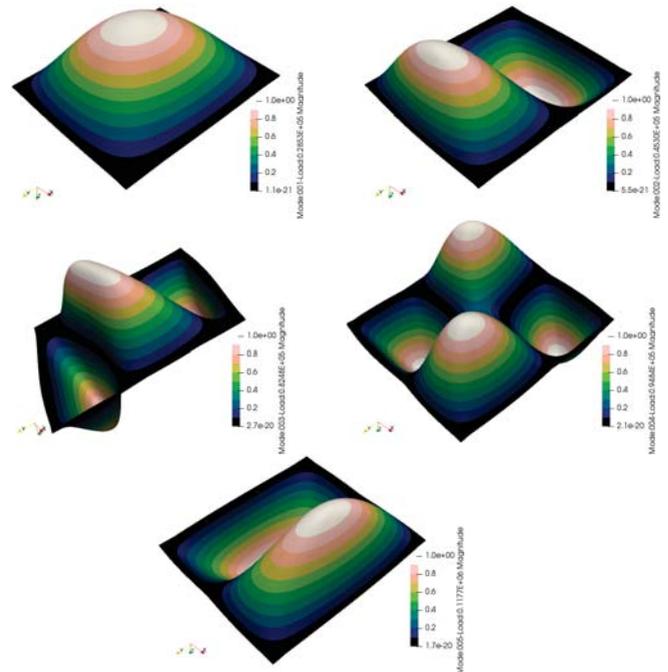

Fig. 2 First five buckling load of optimal VAT design.

4.06% and 2.32% lower number of Degree of Freedom (DOF) respect to the model in ABAQUS. For the ease of optimization and lower computational time, $10B3$ elements are chosen to analyse through the GA. The optimization process is done in a sequence of iteration which is shown in Fig. 1 for the first critical buckling load. As it can be seen, by increasing the number of iteration, the results follow the convergence of the optimum results. The optimum layup design is obtained by $[< 9°|51° >< -9°| - 51° > / < -9°| - 51° >< 9°|51° >]_4$ with $17.39 \ kN$ for the first critical buckling load which is shown 27.67% improvement respect to the FEM reference model. First five buckling modes for optimum results are shown in Fig. 2.
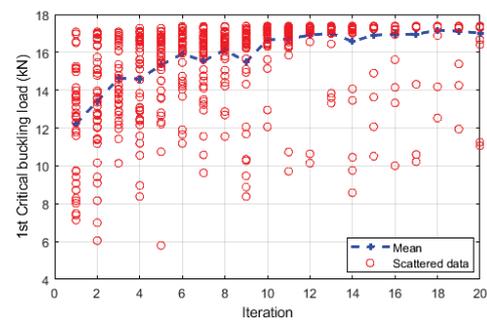

Fig. 1 GA optimization in different iteration and the mean value of each iteration cycle

Besides, through the optimization process, all stochastic T_0, T_1 and corresponding first critical buckling load are collected and showed how each individual parameter is connected and moved through the time of optimization to obtain the optimal result, see Fig. 3. As it is shown in the current figure, the
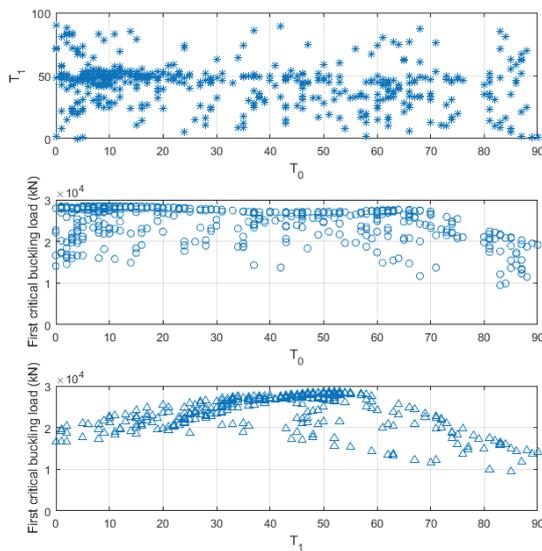
Fig. 3 Distribution of stochastic GA for design variables and first critical buckling load
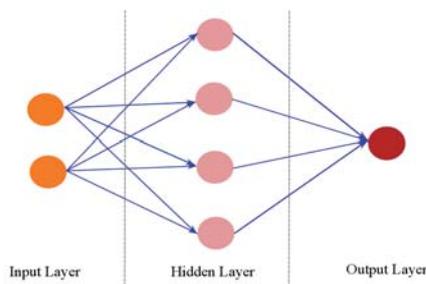

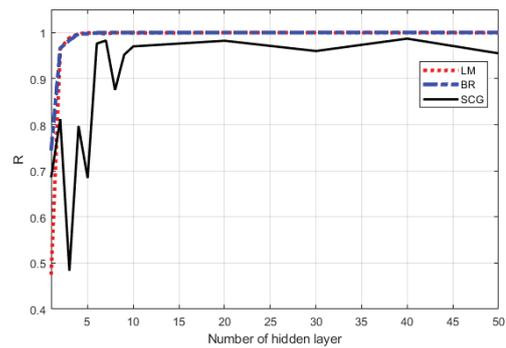Fig. 4 ArchitectureE of Artificial Neural Network

Fig. 5 Comparison between different fit ANN algorithm based on the number of hidden layer for VAT problem
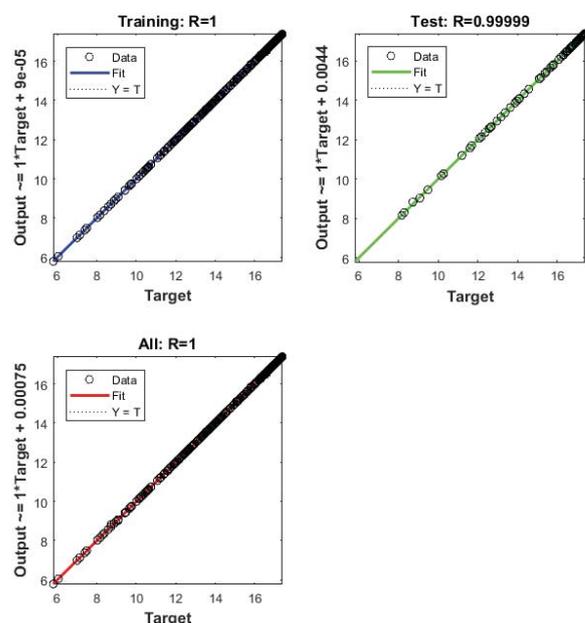
Fig. 6 Neural network training and testing performance

colony of T_0 and T_1 showed the convergence respect to the optimum first buckling load. Most population collection of data have shown a convergence of each parameters (T_0, T_1 and F_{cr}) in Fig. 3.

B. Artificial Neural Network

Materials properties and design prediction are quite different problems, in which the former is usually pointed to as a forward modelling problem and the latter is an inverse design problem [29]. For this purpose, all the data also investigated through the Artificial Neural Network (ANN) framework. A neural network is express as a computational model whose layered structure follows the structure of the network of neurons in the brain, with layers of associated nodes. A neural network can learn from data, therefore it can be trained to recognize patterns, classify data and predict coming events. A neural network consists of an input layer, one or more hidden layers and an output layer, see Fig. 4.

The layers are interconnected via nodes or neurons, with each layer using the output of the previous layer as input.

The quality of VAT optimization affected by design variables is evaluated by current NN. This paper proposes a neural network-based prediction model for detection and prediction critical buckling load as a forward model based on GA. Three different NN functions are used to fit as a model on the VAT problem optimization: Levenberg-Marquardt (LM), Scaled Conjugate Gradient (SCG) [30]. Performance measurement of BR, LM, and SCG algorithms have been analyzed through the NN. The results showed convergence of BR and LM models, while the BR function illustrated the higher value of R Parameter in the start point, see Fig. 5. The R value is an indication of the relationship between the outputs and targets. The R can be changed between 0 and 1, the best value for R is equal to 1. Moreover, the different number of the hidden layer are examined for a different mentioned algorithm to obtain an accurate number of hidden layer for current VAT buckling
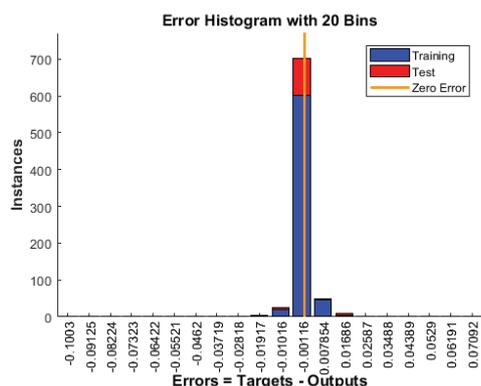
Error Histogram with 20 Bins

Fig. 7 Error histogram for training and testing through the ANN frame

problem. The results are shown by 30 hidden layer through BR algorithm, correct results can be obtained by the minimum error. Based on the 30 number of hidden layer in BR algorithm, three plots of training, testing and all of the results are shown in Fig. 6. The dashed line in each plot represents the perfect result outputs = targets. In current problem dashed line is completely covered by the solid line which represents the best fit linear regression line between outputs and targets, where $R = 1$, is an exact linear relationship between outputs and targets which can be obtained. For this problem, the training data indicates a completely fit and also the test is in a good fit by R indication parameter. In the end, the error diagram for validation and test results is obtained and shown in Fig. 7. The blue bars represent the training data and the red bars are refers to the testing data. Current histogram can show the outliers, which are data points where the fit is significantly worse than the majority of data.

V. CONCLUSION

In the current work, the GA is provided to optimize the buckling load problem of VAT composites by the LW theory in CUF framework. Present VAT laminates built based on LW theory to show the ability of CUF approach to obtain accurate layer by layer composites model. The unified formulation showed the capability to achieve optimum results through the GA optimization procedure. LW model in combination with optimization can be introduced as an accurate model due to eliminating the calculation of the lamination parameters or other extra computation processes to obtain approximation results. The optimum results showed that improvement of the buckling load can be up to 27.67% with respect to VAT_1 in the reference design. The robustness outcomes confirm that the GA is a suitable method to use for the optimum concerning the VAT problem in LW model. The results show that a combination of CUF and GA can be provided as a suitable and reliable method to obtain a robust optimum for optimization of VAT problems in different structural analyses. In the end, ANN can be proved the optimum results and showed the best fit model with 30 number of the hidden layer through the BR algorithm for VAT problem through the buckling load.

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